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M. A./M. Sc. (Previous)

Term End Examination, June-July, 2020-21

MATHEMATICS

Paper First

(Topology)

Time : Three Hours] [*Maximum Marks : 70*

[*Minimum Pass Marks : 14*

Instructions for Candidate :

Section–A : Question Nos. **01** to **08** are very short answer type questions. Attempt all questions. Each question carries 01 mark. Answer each of these questions in **1** or **2** words/**1** sentence.

Section–B : Question Nos. **09** to **14** are half short answer type questions. Attempt any *four* questions. Each question carries $2\frac{1}{2}$ marks. Answer each of these questions in about **75** words or half page.

Section–C : Question Nos. **15** to **18** are short answer type questions. Attempt any *three* questions. Each question carries 05 marks. Answer each of these questions in about **150** words or one page.

Section–D : Question Nos. **19** to **22** are half long answer type questions. Attempt any *two* questions. Each question carries 10 marks. Answer each of these questions in about **300** words or two pages.

Section–E : Question Nos. **23** and **24** are long answer type questions. Attempt any *one* question. Each question carries 17 marks. Answer each of these questions in about **600—750** words or 04—05 pages.

Section—A

1. Define a countable set.
2. What is the derived set of Q, the set of rationals ?
3. Let A = [1, 2) and B = (2, 4]. The what is *d* (A, B) ?
4. State Schwarz inequality for the linear space $\mathbb{C}^n$.
5. A sequence in a Hausdorff space can converge to two distinct points. (True/False)
6. Compact subspaces of Hausdorff spaces are closed. (True/False)
7. State Tychonoff theorem.

8. Every regular space with countable basis is :

   (a) Hausdorff

   (b) Normal

   (c) Disjoint

   (d) None of these

Section—B

9. Define an equivalence relation with an example.
10. Show that every finite set is a closed set.
11. Give *two* different metrics for the set of real numbers.
12. Define basis on a topological space.
13. Prove that closed subspaces of compact spaces are compact.
14. Show that compact Hausdorff spaces are normal.

Section—C

15. State and prove triangle inequality for complex numbers.
16. Show that Cantor's set Γ is non-dense and perfect.
17. Show that C $[a, b]$, the set of all real valued continuous functions defined on $[a, b]$ with the following metric :

$$d(f, g) = \text{Sup} \ \left| f(x) - g(x) \right| : x \in [a, b]$$

$$\forall \ f, g \in C[a, b]$$

   is complete.

18. Prove that linearly ordered spaces are normal.

Section—D

19. State and prove Balzano-Weierstrass theorem.

20. Prove that all completions of a metric space are isometric.

21. State and prove Banach fixed point theorem.

22. State and prove Urysohn lemma.

Section—E

23. (a) Show that the set $\mathbb{C}$ of all complex numbers is a metric space under :

$$d(x, y) = \frac{|x + y|}{\left(1 + |x|^2 \quad 1 + |y|^2\right)^{1/2}}$$

(b) Give an example of a pseudometric which is not a metric.

24. State and prove Tietze extension theorem.

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M. A./M. Sc. (Previous)
Term End Examination, June-July, 2020-21

MATHEMATICS

Paper Second

(Real Analysis)

Time : Three Hours] [*Maximum Marks : 70*

[*Minimum Pass Marks : 14*

Instructions for Candidate :

Section–A : Question Nos. **01** to **08** are very short answer type questions. Attempt all questions. Each question carries 01 mark. Answer each of these questions in **1** or **2** words/**1** sentence.

Section–B : Question Nos. **09** to **14** are half short answer type questions. Attempt any *four* questions. Each question carries $2\frac{1}{2}$ marks. Answer each of these questions in about **75** words or half page.

Section–C : Question Nos. **15** to **18** are short answer type questions. Attempt any *three* questions. Each question carries 05 marks. Answer each of these questions in about **150** words or one page.

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Section–D : Question Nos. **19** to **22** are half long answer type questions. Attempt any *two* questions. Each question carries 10 marks. Answer each of these questions in about **300** words or two pages.

Section–E : Question Nos. **23** and **24** are long answer type questions. Attempt any *one* question. Each question carries 17 marks. Answer each of these questions in about **600—750** words or 04—05 pages.

Section—A

1. If p is prime, then :

   (a) $\sqrt{p}$ is rational

   (b) $\sqrt{p}$ is irrational

   (c) $\sqrt{p}$ is complex

   (d) None of these

2. Every absolutely continuous function is :

   (a) Constant

   (b) Continuous

   (c) Absolutely discontinuous

   (d) None of these

3. If f is R-integrable on $[a, b]$, then :

   (a) $\left|\int_a^b f(x)\,dx\right| \le \int_a^b |f(x)|\,dx$

(b) $\left|\int_a^b f(x)\,dx\right| \geq \int_a^b |f(x)|\,dx$

(c) $\left|\int_a^b f(x)\,dx\right| = \int_a^b |f(x)|\,dx$

(d) None of these

4. The sequence $\left\{\frac{(-1)^n}{n}\right\}$ is :

(a) Convergent at 0

(b) Divergent

(c) Oscillatory

(d) None of these

5. Triangular inequality states "If X and Y are in R", then :

(a) $|X + Y| \leq |X| + |Y|$

(b) $|X + Y| \geq |X| + |Y|$

(c) $|X + Y| = |X| + |Y|$

(d) None of these

6. The inverse of a square matrix A exists iff :

(a) $A = 0$

(b) $|A| = 0$

(c) $|A| \neq 0$

(d) adj $A \neq 0$

7. The value of :

$$\int_0^1 \int_0^1 (x^2 + y^2)\, dx\, dy$$

is :

(a) 0

(b) $\frac{1}{3}$

(c) $\frac{2}{3}$

(d) 1

8. The value of :

$$\int_0^1 \int_0^{\sqrt{1-y^2}} y\, dy\, dx$$

is :

(a) $\frac{1}{3}$

(b) $\frac{2}{3}$

(c) 0

(d) 1

Section—B

9. State Bolzano-Weierstrass theorem.
10. State mean value theorem.
11. Explain absolute and conditional convergence.
12. Define inner product space.
13. Define linear transformation.
14. Define the sets with zero content.

Section—C

15. If g is continuous at $x_0, g(x_0)$ is an interior point of Df, and f is continuous at $g(x_0)$. Then prove that $f \text{ o } g$ is continuous at x_0.

16. If f is bounded on [a, b], then prove that :

$$\int_{\underline{a}}^{b} f(x)\, dx \leq \int_{a}^{\overline{b}} f(x)\, dx$$

17. If :

$$f : \mathrm{R}^2 \to \mathrm{R}$$

is defined as follows :

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2}; & (x, y) \neq (0, 0) \\ 0 \quad ; & (x, y) = (0, 0) \end{cases}$$

then find :

$$\lim_{(x,y)\to(0,0)} f(x, y)$$

18. Find the value of :

$$\int_0^1 \int_0^{\sqrt{1+x^2}} \frac{dx\, dy}{1 + x^2 + y^2}$$

Section—D

19. If f is continuous on a closed and bounded interval [a, b], then prove that f is uniformly continuous on [a, b].

20. If f is integrable on [a, b], then prove that :

$$\int_{\underline{a}}^{b} f(x)\, dx = \int_{a}^{\overline{b}} f(x)\, dx = \int_{a}^{b} f(x)\, dx$$

21. If f is continuous on a compact set S in R^n, then prove that f is bounded on S.

22. Find the area of the set bounded by the curves :

$$y = x^2 + 9$$

$$y = x^2 - 9$$

$$x = -1$$

$$x = 1$$

Section—E

23. Determine the convergence of the following series :

(a) $\sum \sin \frac{\pi}{n^2}$

(b) $\sum \frac{1}{n} \tan \frac{\pi}{n}$

24. State and prove Cauchy's uniform convergence criterion.

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M. A./M. Sc. (Previous)

Term End Examination, June-July, 2020-21

MATHEMATICS

Paper Third

(Partial Differential Equation)

Time : Three Hours] [*Maximum Marks : 70*

[*Minimum Pass Marks : 14*

Instructions for Candidate :

Section–A : Question Nos. **01** to **08** are very short answer type questions. Attempt all questions. Each question carries 01 mark. Answer each of these questions in **1** or **2** words/**1** sentence.

Section–B : Question Nos. **09** to **14** are half short answer type questions. Attempt any *four* questions. Each question carries $2\frac{1}{2}$ marks. Answer each of these questions in about **75** words or half page.

Section–C : Question Nos. **15** to **18** are short answer type questions. Attempt any *three* questions. Each question carries 05 marks. Answer each of these questions in about **150** words or one page.

Section–D : Question Nos. **19** to **22** are half long answer type questions. Attempt any *two* questions. Each question carries 10 marks. Answer each of these questions in about **300** words or two pages.

Section–E : Question Nos. **23** and **24** are long answer type questions. Attempt any *one* question. Each question carries 17 marks. Answer each of these questions in about **600—750** words or 04—05 pages.

Section—A

1. Complete integral of $p + q = pq$ is :

   (a) $z = ax + \dfrac{ay}{a-1}$

   (b) $z = ax + by + c$

   (c) $z = ax + \dfrac{ay}{a-1} + c$

   (d) $z = \dfrac{1}{a^x} + \dfrac{ay}{a-1}$

2. Partial differential equation $pq = 1$ is of the following standard form :

   (a) I

   (b) II

   (c) III

   (d) IV

3. Solution of PDE :

$$s + p - q = z + xy$$

   is :

   (a) $z = e^{-z} f_2(y) + x - y + xy + 1$

   (b) $z = e^{z} f_1(y) + e^{-y} f_2(x) - xy - y + x + 1$

   (c) $z = e^{x} f_1(y) + xy - y + x + 1$

   (d) None of the above

4. A surface passing through the two lines :

$$y = 0 = z$$

$$y = 3 = z$$

   satisfying the PDE :

$$\frac{\partial^2 z}{\partial y^2} = 12x^2 y$$

   is :

   (a) $z = y(1 - 18x^2) + 2y^3x^2$

   (b) $z = x(1 - 18y^2) + 3x^2y^2$

   (c) $z = 2y(1 - 9x^2) + y^3x^2$

   (d) None of the above

5. The equation :

$$\nabla^2 V = -4\pi G G \rho$$

is :

(a) Green's equation

(b) Laplace's equation

(c) Poisson's equation

(d) Gauss' equation

6. Heat equation :

$$\frac{\partial u}{\partial t} = k \, , \frac{\partial^2 u}{\partial x^2}$$

is also known as :

(a) Diffusion equation

(b) Poisson's equation

(c) Gauss' equation

(d) Green's equation

7. Classify the wave equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{c^2} \frac{\partial u}{\partial t}$$

(a) Parabolic

(b) Elliptic

(c) Hyperbolic

(d) None of the above

8. A delta function of strength 0, is :

(a) 1

(b) Null

(c) Infinity

(d) None of the above

Section—B

9. Solve :

$$p^2 + q^2 = 1$$

10. Solve :

$$25r - 40s + 16t = 0$$

11. Show that if a harmonic function vanishes everywhere on the boundary, then it is identically zero everywhere.

12. Find the deflection $u(x, t)$ of the vibrating string whose length is π^2 and $c^2 = 1$ corresponding to zero initial velocity and initial deflection :

$$f(x) = k(\sin x - \sin 2x)$$

13. Define Green's function of second kind.

14. Find the Green's function of the equation $\nabla^2 u = 0$ for the upper half plane.

Section—C

15. Solve :

$$q^2 y^2 = z(z - px)$$

16. Solve :

$$(D^2 + 3DD' + 2D'^2)\, z = x + y$$

17. Solve by the method of separation of variables :

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

18. State and Prove "The Maximum-Minimum Principle."

Section—D

19. Find the complete integral of :

$$(x^2 - y^2)\, pq - xy\,(p^2 - q^2) - 1 = 0$$

20. Solve :

$$4r - 4s + t = 16 \log (x + 2y)$$

21. Find the steady state temperature distribution in a rectangular plate of sides a and b insulated at the lateral surface and satisfying the boundary conditions :

$$u\,(0, y) = u\,(a, y) = 0 \qquad \text{for } 0 \le y \le b$$

and $u\,(x, b) = 0$ and $u\,(x, 0) = x\,(4 - x)$ for $0 \le y \le a$

22. Find the solution of the wave equation :

$$\frac{\partial^2 y}{dt^2} = C^2 \frac{\partial^2 y}{\partial x^2}$$

such that $y = p_0 \cos pt,$ when $x = 1$ and $y = 0$ when $x = 0$ (b_0 is a constant).

Section—E

23. State and prove “First Meant Value Theorem”.

24. Find the potential u inside :

$$0 \le r \le a$$

$$0 \le \theta \le 2\pi$$

$$0 \le z \le h$$

if the potential of the top $z = h$ and on the lateral surface $r = a$ is held at zero, while on the base $z = 0$ the potential is given by :

$$u\,(r, \theta, 0) = V_0\left(1 - \frac{r^2}{a^2}\right)$$

where V_0 is a constant.

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M. A./M. Sc. (Previous)
Term End Examination, June-July, 2020-21

MATHEMATICS

Paper Fourth

(Discrete Mathematics)

Time : Three Hours] [*Maximum Marks : 70*

[*Minimum Pass Marks : 14*

Instructions for Candidate :

Section–A : Question Nos. **01** to **08** are very short answer type questions. Attempt all questions. Each question carries 01 mark. Answer each of these questions in **1** or **2** words/**1** sentence.

Section–B : Question Nos. **09** to **14** are half short answer type questions. Attempt any *four* questions. Each question carries $2\frac{1}{2}$ marks. Answer each of these questions in about **75** words or half page.

Section–C : Question Nos. **15** to **18** are short answer type questions. Attempt any *three* questions. Each question carries 05 marks. Answer each of these questions in about **150** words or one page.

Section–D : Question Nos. **19** to **22** are half long answer type questions. Attempt any *two* questions. Each question carries 10 marks. Answer each of these questions in about **300** words or two pages.

Section–E : Question Nos. **23** and **24** are long answer type questions. Attempt any *one* question. Each question carries 17 marks. Answer each of these questions in about **600—750** words or 04—05 pages.

Section—A

1. $\sim(\sim p \wedge q) =$ ....
2. Draw the symbol for NAND gate ....
3. In a set theory $n(A \cup B) =$ ....
4. Every subset of a countable set is ....
5. (G, *) is said to be groupoid if ....
6. Let H be a subgroup of G and $a \in G$ be any arbitrary element, then $(Ha)^{-1} =$ ....
7. If $a, b \in R$ (Ring), then $(a+b)^2 =$ ....
8. Any superset of linearly dependent set is always ....

Section—B

9. Define Tautology and Contradiction.
10. In set theory, prove that :

$$A - (A - B) = A \cap B$$

11. Define Domain and Range of relation.
12. Define Cosets.
13. Define singular and non-singular linear transformation.
14. Define universal quantifier.

Section—C

15. Express the following Boolean function in conjunctive normal form :

$$f(x, y, z) = (xy' + xz)' + x'$$

16. Prove that in set theory :

$$(A \Delta B) \Delta C = A \Delta (B \Delta C)$$

17. Prove that the fourth root of unity is an Abelian group with the operation multiplication.

18. Show that the mapping $T : V_2 \to V_2$ defined on $V_2(R)$ as :

$$T(x, y) = (2x + 3y, 3x - 4y)$$

is linear transformation.

Section—D

19. If Ayush has completed BE or MBA even, then he is assured of a good job. If Ayush is assured of good job he is happy. Ayush is not happy so Ayush has not completed MBA and BE. Check the validity of the above agreement.

20. Let $f : X \to Y$ be any arbitrary mapping defined from X to Y. Let A, B be any subsets of X. Then prove that :

$$f(A \cup B) = f(A) \cup f(B)$$

21. Let I be the set of integrals. Let * be the binary operation defined by :

$$a * b = a + b + 1 \ \forall \ a, b \in I$$

Show that (I, *) is an Abelian group.

22. Prove that the Left (Right) modulo H in group (G, *) is an equivalence relation.

Section—E

23. Show that the vectors (1, 0, 0); (1, 1, 0) and (1, 1, 1) is a basis for R^3. Also express standard basis of R^3 in term of basis S.
24. State and prove Lagrange's theorem.